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STABILITY OF A LIQUID FILM ON THE SURFACE OF A ROTATING CYLINDER

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ABSTRACT: The present article examines the stability of a thin layer of a viscous liquid held by forces of surface tension on the surface of an infinite rotating cylinder. We show that the cylindrical figure of the liquid layer is unstable under rotation, no matter how slow the latter is, with respect to long-wave annular axisymmetric disturbances. /98*

In an undisturbed motion of the film, only the azimuthal component of the velocity is nonzero (rotation of a solid):

$$V_\varphi = \Omega r, \quad P = \frac{\rho\Omega^2 r^2}{2} - \frac{\rho\Omega^2 (R+h)^2}{2} + \frac{\alpha}{R+h} \tag{1}$$

Here, Ω is the angular velocity of rotation of the cylinder (directed along the axis of the cylinder, i.e., the z-axis), P is the pressure, R is the radius of the solid cylinder, h is the thickness of the film, and α is surface tension.

Suppose that the perturbations of the free surface of the film hf(z, t) and of the flow are axisymmetric:

$$\begin{gathered} v_r = u(r)\, e^{ikz+\sigma t}, \qquad V_\varphi + v(r)\, e^{ikz+\sigma t} \\ v_z = w(r)\, e^{ikz+\sigma t}, \qquad p = p(r)\, e^{ikz+\sigma t} \\ R + h + hf(z,t) = R + h + h\varepsilon e^{ikz+\sigma t}, \quad \partial(\ldots)/\partial\varphi \equiv 0, \quad \varepsilon \ll 1 \end{gathered} \tag{2}$$

Let us consider Navier-Stokes equations linearized with respect to the perturbation and the continuity

$$\begin{gathered} \sigma u - 2\Omega v = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\frac{d^2u}{dr^2} - k^2u + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2}\right] \\ \sigma v + 2\Omega u = \nu\left[\frac{d^2v}{dr^2} - k^2v + \frac{1}{r}\frac{dv}{dr} - \frac{v}{r^2}\right] \\ \sigma w = -\frac{ik}{\rho}p + \nu\left[\frac{d^2w}{dr^2} - k^2w + \frac{1}{r}\frac{dw}{dr}\right] \end{gathered} \tag{3}$$

$$\frac{du}{dr} + \frac{u}{r} + ikw = 0 \tag{4}$$

*Numbers in the margin indicate pagination in the foreign text.

The boundary conditions are the following:

For r = R,

$$u = v = W = 0 \tag{5}$$

On the free surface,

$$\sigma_{ik}n_k = -\alpha\left(\frac{1}{R_1} + \frac{1}{R_2}\right)n_i, \qquad v_r = \frac{d}{dt}(hf) = \frac{\partial(hf)}{\partial t} \tag{6}$$

The stress tensor σ_{ik} and the unit vector n_i are taken in a cylindrical coordinate system [1].

Here, $n_\varphi = 0$ by virtue of the axial symmetry of the perturbation and $n_z =$ $= -hdf/dz$ is of the same order as the pertubation. The expression for the principal radii of curvature is also linearized with respect to the perturbation: /99

$$R_1 = R + h + hf(z, t), \qquad R_2^{-1} = -h\frac{\partial^2 f}{\partial z^2} \tag{7}$$

Keeping this in mind, we obtain the boundary conditions on the free surface

$$\begin{gathered} -\Omega^2(R+h)h\varepsilon - \frac{p}{\rho} + 2\nu\frac{du}{dr} = \frac{\alpha}{\rho}\left(\frac{h\varepsilon}{(R+h)^2} - hk^2\varepsilon\right) \\ \frac{dv}{dr} - \frac{v}{r} = 0, \qquad \frac{dw}{dr} + iku = 0, \qquad u = \sigma h\varepsilon \end{gathered} \tag{8}$$

In the case of a thin film, we set $h/R \ll 1$ and we introduce the new variable

$$r = R + hx \tag{9}$$

Then,

$$\frac{d}{dr} = \frac{1}{h}\frac{d}{dx}, \qquad \frac{1}{r} = \frac{1}{R} + O\left(\frac{h}{R}\right) \tag{10}$$

Treating h/R as a small parameter, let us expand (3)-(8) with respect to it. As a zeroth approximation, we obtain, after elimination of pressure p and w(r),

$$\begin{gathered} \left[\frac{d^2}{dx^2} - \left(\frac{\sigma h^2}{\nu} + k^2h^2\right)\right]\left[\frac{d^2}{dx^2} - k^2h^2\right]u = \frac{2\,\Omega k^2h^4}{\nu}v \\ \left[\frac{d^2}{dx^2} - \left(\frac{\sigma h^2}{\nu} + k^2h^2\right)\right]v = \frac{2\,\Omega h^2}{\nu}u \\ u = v = \frac{du}{dx} = 0 \quad \text{at } x = 0 \end{gathered} \tag{11}$$

$$x = 1, \quad \left[\frac{d^2}{dx^2} - \left(\frac{\sigma h^2}{\nu} + k^2 h^2\right)\right]\frac{du}{dx} - 2k^2h^2\frac{du}{dx} + \frac{k^2h^3}{\sigma\nu}\left(\Omega^2 R - \frac{\alpha}{\rho}k^2\right)u = 0$$

$$\frac{d^2u}{dx^2} + k^2h^2u = 0, \qquad \frac{dv}{dx} = 0$$

Let us look at long-wave perturbations. If we set

$$kh \ll 1, \qquad \frac{\sigma h^2}{\nu} \ll 1, \qquad \frac{\Omega h^2}{\nu} \ll 1$$

expand the solution in terms of the small parameters kh, $\Omega h^2/\nu$, $\sigma h^2/\nu$, and confine ourselves to the principal terms of the expansion, we obtain from (11)

$$u(x) = A_0 + A_1 x + A_2 x^2 + A_3 x^3 + O\left(\frac{\Omega^2 h^4}{\nu^2},\ k^2h^2,\ \frac{\sigma h^2}{\nu}\right) \tag{12}$$

When we substitute u(x) into the boundary conditions (11), we obtain for σ the secular equation

$$\det \|a_{ij}\| = 0 \quad (ij = 1, 2)$$
$$a_{11} = \frac{k^2h^3}{\sigma\nu}\left(\Omega^2 R - \frac{\alpha}{\rho}k^2\right) + O\left(\frac{\Omega^2 h^1}{\nu^2},\ k^2h^2\right), \qquad a_{21} = 2 + O\left(\frac{\Omega^2 h^4}{\nu^2},\ k^2h^2\right)$$
$$a_{12} = \frac{k^2h^3}{\sigma\nu}\left(\Omega^2 R - \frac{\alpha}{\rho}k^2\right) + 6 + O\left(\frac{\Omega^2 h^4}{\nu^2},\ k^2h^2\right), \qquad a_{22} = 6 + O\left(\frac{\Omega^2 h^4}{\nu^2},\ k^2h^2\right) \tag{13}$$

From this we obtain

$$\sigma = \frac{k^2h^3}{3\nu}\left(\Omega^2 R - \frac{\alpha}{\rho}k^2\right)\left[1 + O\left(\frac{\Omega^2 h^4}{\nu^2},\ k^2h^2,\ \frac{\sigma h^2}{\nu}\right)\right] \tag{14}$$

Thus, a cylindrical film is unstable with respect to perturbations with wave number

$$k < \left(\frac{\rho\Omega^2 R}{\alpha}\right)^{1/2}$$

REFERENCES

1. Landau, L. D. and Lifshitz, E. M. Mekhanika sploshnykh sred (Mechanics of Continuous Media), Moscow, Gostekhizdat, 1953.